

# An Assimilation of TOPEX/POSEIDON Data

Ichiro Fukumori, Ramanujam Raghunath, Lee-Lueng Fu, and Yi Chao

*Jet Propulsion Laboratory, California Institute of Technology*

~~(submitted for the 1998 JGR-WOCE Report)~~

Data assimilation provides a means to synthesize diverse observations in a dynamical framework and thereby estimate the entire three-dimensional state of the ocean. The goodness of such estimate, however, depends on assumptions underlying the solution. In principle, given correct assumptions, an assimilation necessarily leads to a statistically more accurate model estimate than one without assimilation. Improvements result in every aspect of the solution (or no worsening should occur when there is not enough information), even for properties not directly measured.

This report summarizes progress in a recent study of assimilating altimetric data into a global ocean general circulation model. The accuracy and consistency of the calculation are discussed, so as to ascertain the soundness of the assumptions (and therefore its results) and to illustrate some of the issues in attaining a more comprehensive estimate.

The study employs a global model with a modest resolution ( $2^\circ$  zonal,  $1^\circ$  meridional, and 12 vertical levels) extending from  $80^\circ\text{S}$  to  $80^\circ\text{N}$ . The model is based on the Modular Ocean Model (MOM) of the Geophysical Fluid Dynamics Laboratory (GFDL), and is forced by daily National Center for Environmental Prediction (NCEP) wind analyses and the Comprehensive Ocean-Atmosphere Data Set (COADS) heat fluxes. Three-years (1993-95) of sea level anomalies measured by TOPEX/POSEIDON (T/P) are assimilated into the model using the model's time-mean sea level as a reference surface. The data are assimilated continuously along the satellite's ground tracks using a time-asymptotic, reduced-state Kalman filter and smoother (Fukumori et al., 1993; Fukumori and Malanotte-Rizzoli, 1995). Amplitudes of the barotropic and first baroclinic modes, which are the leading components of the global large-scale wind-driven sea level variability (Fukumori et al., 1998), make up the reduced state on a  $10^\circ$  by  $5^\circ$  grid.

Prior uncertainties of model and data, which in essence define the assimilation problem, were estimated by the so-called covariance matching method (Fu et al., 1993; Menemenlis and Chechelnitsky, 1998). Model process error is modeled in the form of wind forcing. Figure 1 illustrates the consistency of the resulting assimilation by comparing actual and theoretical estimates of improvement in model explained T/P sea level variance. The comparison is made based on the innovation sequence, viz., data not yet assimilated, and therefore the reduction in model-data residuals (positive numbers in Figure 1a) is not trivial. Figure 1a shows that improvements are achieved almost everywhere but with spatially varying magnitude reflecting inhomogeneity of the physics and its uncertainties. The results are comparable in amplitude and spatial structure with theoretical expectations (Figure 1b), demonstrating the first-order consistency of the calculation.

The reliability of the estimate can be assessed further by comparisons with independent observations, namely data that are withheld from assimilation. Figure 2 shows examples of such comparisons between model estimates and in situ measurements of sea level, subsurface temperature and velocity, and bottom pressure. In each example, the altimetric assimilation (blue) is closer to independent measurements (black) than without assimilation (red), in rough agreement with formal error estimates also shown in the figure. The T/P assimilation in effect corrects amplitudes of events that are inadequately simulated by model alone. The limited correction for the ultra high frequency oscillations in bottom pressure (periods less than 10-days) is due primarily to the sampling frequency of the altimeter.

While majority of comparisons show similar improvements, thereby validating the assimilation, some result in little improvement at all (Figure 3). However, most of these can be attributed to physics missing from the model, rather than a failure of the assimilation per se. The point is clarified by identifying what is solved by assimilation (Cohn, 1997).

Observations ( $\mathbf{y}$ ) are in essence some function ( $\mathbf{E}$ ) of the true oceanic state ( $\mathbf{w}$ ) plus some errors in making the observations ( $\epsilon$ ), such as instrument error;

$$\mathbf{y} = \mathbf{E}(\mathbf{w}) + \epsilon \quad (1)$$

In terms of the true model state ( $\mathbf{x}$ ), this can be rewritten as,

$$\mathbf{y} = \mathbf{H}(\mathbf{x}) + \epsilon + \{\mathbf{E}(\mathbf{w}) - \mathbf{H}(\mathbf{x})\} \quad (2)$$

where  $\mathbf{H}$  is the model equivalent of  $\mathbf{E}$ . The last term in (2) is called model representation error and is the difference between the real ocean and what the model can resolve. Assimilation is in effect the inversion of the first term  $\mathbf{H}$  in (2). For such inversion, model representation errors are indistinguishable from instrument errors and cannot be corrected by assimilation. Representation errors typically are processes with scales smaller than model resolution. In the present altimetric assimilation, representation errors largely correspond to variabilities due to meso-scale eddies.

In Figure 3, the dominant signal in the data is incoherent over 400 m depth and is qualitatively lacking in the model, suggestive of it being something the model cannot simulate, i.e., model representation error. The model is either too coarse in the vertical (there are only two levels between 125 and 500 m) or lacking in some other physics to adequately simulate temperature variability at this location. The formal error estimate does not include representation error and is therefore much smaller than actual model-data differences.

Model-data discrepancies increase in a few examples indicating possible inconsistencies in the calculation. In particular, comparison with XBT data (W.White, personal communication) shows significantly larger residuals at 300m depth along the equator in the Pacific Ocean. Figure 4 compares subsurface temperatures at a particular location on the equator. Unlike Figure 3, much of the signal is coherent with the model indicating insignificant representation errors. Worsening by assimilation (Figure 4b) begins about day 1100, and is of opposite sign relative to simulation between 140m

and 300m depths. Analysis suggests that these differences arise due to advection of opposite mean meridional temperature gradients between these two depths. The approximate filter assumes a closed system within the reduced state. These discrepancies, however, indicate that close to the equator the physics significantly depends on shorter vertical scales than those retained in the present filter, violating this assumption.

Results presented above illustrate some of the issues of assimilation by demonstrating measures of consistency and accuracy. In particular, distinction of model representation error is critical in understanding what can be achieved by assimilating data and optimizing the results. Work is in progress to improve the estimates by correcting inaccuracies identified in both filter and model and by assimilating other data types including those withheld from the present analysis.

## References

- Cohn, S. E., 1997. An introduction to estimation theory, *Journal of the Meteorological Society of Japan*, **75**, 257–288.
- Fu, L.-L., I. Fukumori and R. N. Miller, 1993. Fitting dynamic models to the Geosat sea level observations in the Tropical Pacific Ocean. Part II: A linear, wind-driven model, *Journal of Physical Oceanography*, **23**, 2162–2181.
- Fukumori, I., J. Benveniste, C. Wunsch and D. B. Haidvogel, 1993. Assimilation of sea surface topography into an ocean circulation model using a steady-state smoother, *Journal of Physical Oceanography*, **23**, 1831–1855.
- Fukumori, I., and P. Malanotte-Rizzoli, 1995. An approximate Kalman filter for ocean data assimilation; An example with an idealized Gulf Stream model, *Journal of Geophysical Research*, **100**, 6777–6793.
- Fukumori, I., R. Raghunath, and L. Fu, 1998. The nature of global large-scale sea level variability in relation to atmospheric forcing: A modeling study, *Journal of Geophysical Research*, **103**, 5493–5512.
- Menemenlis, D., and M. Chechelnitsky, 1998. Error estimates for an ocean general circulation model from altimeter and acoustic tomography data, (submitted to Monthly Weather Review).

## Figures Captions

Figure 1: Model improvement in simulating observed TOPEX/POSEIDON sea level variability by the assimilation (a), and its theoretical expectation (b). Unit in cm. The values are the root-mean-square difference between the model-data residual variance of the simulation and that of the assimilation (innovation sequence). (The sign is that of simulation residual minus assimilation residual.) Positive (negative) values indicate improvements over (worsening from) simulation.

Figure 2: Comparisons of T/P assimilation (blue) and model simulation (red) with in situ measurements (black); (a) sea level (tide gauge at Christmas Island, 2°N,

157°W), (b) temperature (Tropical Atmosphere Ocean [TAO] array, 200m, 8°N, 180°E), (c) east-west velocity (TAO, 120m, 0°N, 110°W), (d) bottom pressure (Crozet Island, 3600m, 47°S, 52°E). The values are temporal anomalies of the respective estimates. The bars denote formal standard error estimates computed by the Kalman filter.

Figure 3: Temperature anomalies at 2°S 165°E at 125m (a) and 500m (b). Different curves are as in Figure 2. In situ measurements are obtained from a TAO mooring.

Figure 4: Temperature anomalies at 0°N 140°W at 140m (a) and 300m (b). Different curves are as in Figure 2. In situ measurements are obtained from a TAO mooring.

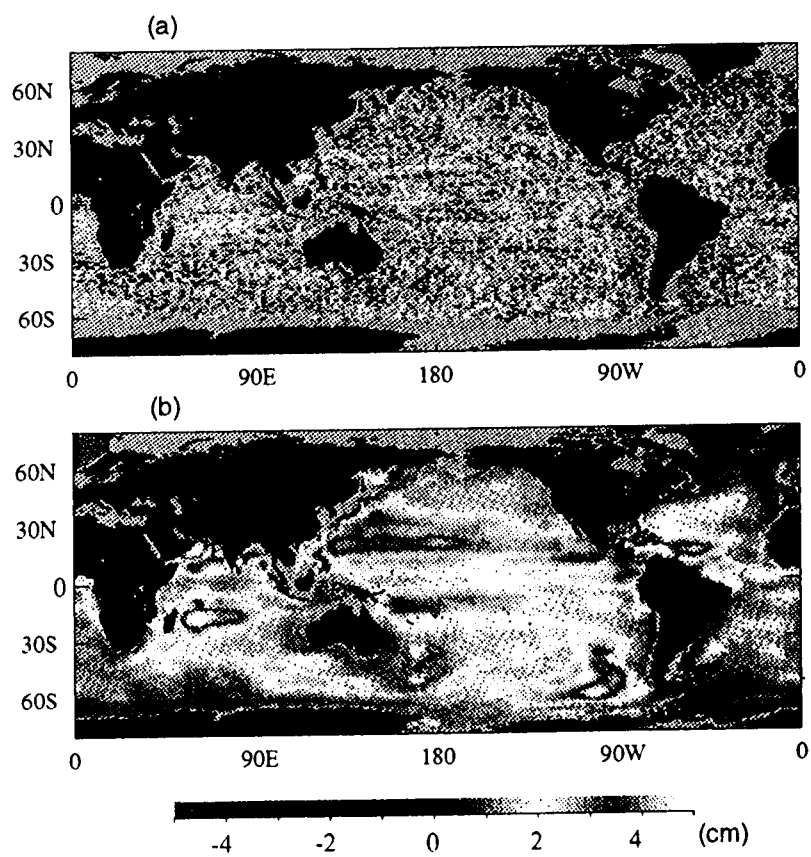


Fig. 1

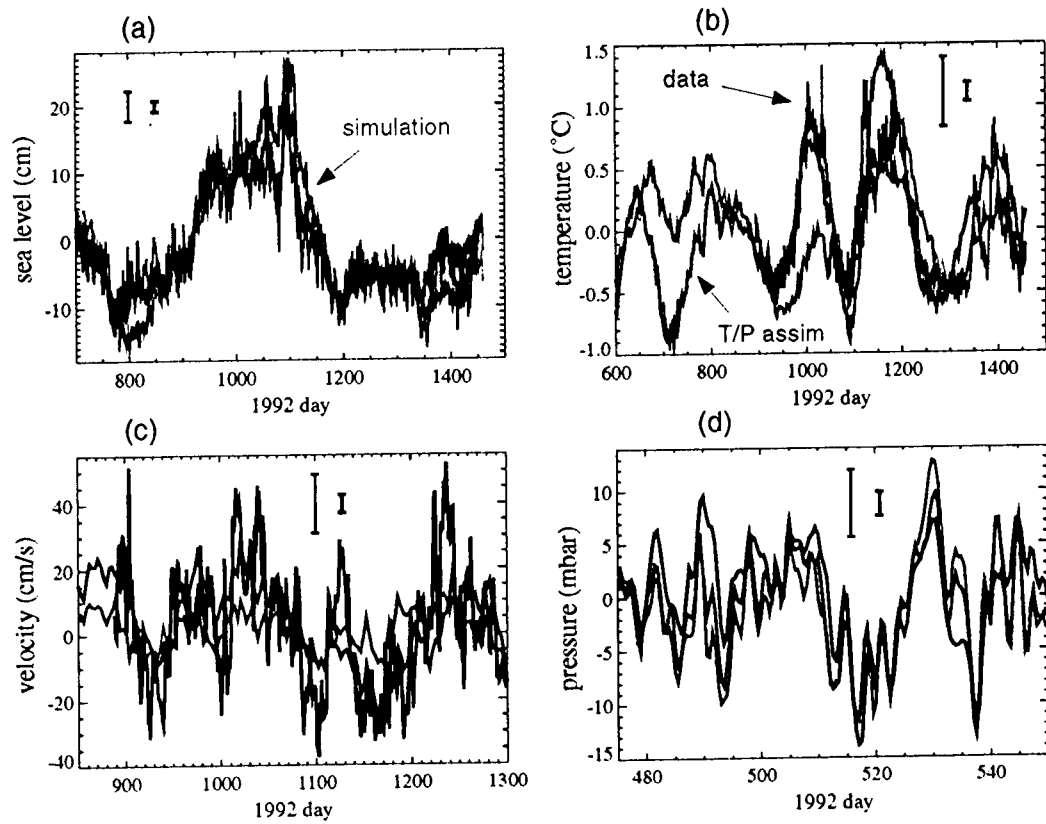


Fig. 2

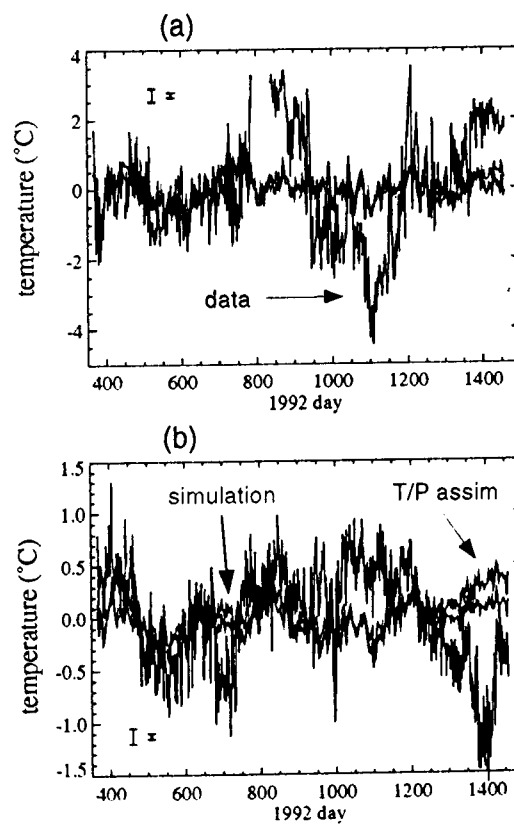


Fig. 3

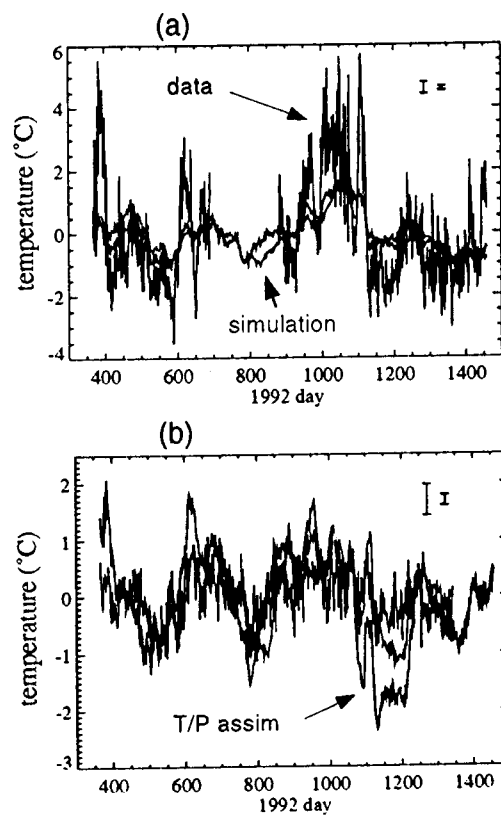


Fig. 4